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Book Review: *Elements of the Random Walk: An Introduction for Advanced Students and Researchers*

Elements of the Random Walk: An Introduction for Advanced Students and Researchers. J. Rudnick and G. Gaspari, Cambridge University Press. 2004.

Had I not found much of interest in this text, I would not have reviewed it. But I did and I am. The book is clearly meant for advanced graduate students in physics rather than in mathematics, since much of the material covered in it requires a grasp of sophisticated techniques known to students who have taken one or more courses in statistical mechanics but which are not generally covered in introductory courses in mathematics.

Roughly speaking, the book falls into two parts, the first containing material that overlaps with parts of my own book (G.H. Weiss, Aspects and Applications of the Random Walk (North-Holland, 1994)) and some of the monograph by Barry Hughes (Random Walks and Random Environments, Oxford University Press, 1995). That is to say, the first part focuses on methods most readily handled by generating functions. These mainly apply especially to properties of Markovian random walks. The second part of the text discusses more advanced topics, path integrals, scaling and renormalization techniques, together with an extensive account of diagrammatic methods as applied to random walk problems.

Moving from generalities to some more specific comments, I think that it might have been useful to at least define the difference between Markovian and non-Markovian models. An account of this distinction might have shed a little more light on the class of generalizations available through the use of CTRW's. Some of the discussion of asymptotic properties of generating functions and the CTRW could have been simplified by a more systematic exposition of Abelian and Tauberian theorems early on. A rather interesting touch in the second chapter on generating functions is a comment on the recurrence property for random walks in which the dimensionality is between 2 and 4. A feature of note in the derivation of boundary conditions is that it is based on electrostatic analogies, rather than being a solely mathematical exposition.

The authors discuss, among other topics in a chapter on variations of the random walk, that of persistent random walks, making elegant use of bra and ket operators in their derivation of two- and three-dimensional telegrapher's equations. They do not mention the vexing question of establishing proper boundary conditions for the telegrapher's equation, a necessity in certain applications not much addressed in the literature. Their exposition of the CTRW could have benefitted from a preliminary introduction to the simplest parts of renewal theory.

The succeeding chapter covers the very interesting topic of defining and calculating what the authors refer to as the shape of a random walk. This is suggested by a study by Solc and Stockmayer to account for the seemingly unintuitive asymmetry in a random walk model of a polymer, in which all of the defining parameters are isotropic. In three dimensions, there are three axes to be accounted for. Once one of the axes is identified as being the largest the distribution of the lengths of the remaining two becomes a conditional one, which overrides the property of symmetry. While there are a number of measures of asymmetry the authors choose the ellipsoid whose elements are the radii of gyration. The asymmetry is reflected in the non-identity of lengths of the axes of the ellipsoid. The authors' analysis of this problem is elegant, and introduces diagrammatic methods used so effectively in different forms when applied to the theory of self-avoiding random walks.

The remaining six chapters of the book deal with what I consider to be more modern aspects of random walk theory. The topics included under this heading are exemplified by self-avoiding walks, whose analysis requires path integrals, diagrammatic methods, scaling arguments, renormalization methods, and fractals. For the most part these topics are handled very simply and elegantly and are certainly an excellent starting point for those wanting a relatively simple and straightforward introduction to these topics. I recommend, in particular, the several sections that deal with diagrammatic methods which are discussed at some length.

The one topic, which is introduced, but would need more fleshing out, is that of fractals, which includes only a cursory discussion of deterministic fractals. The three deficiencies that I found most noticeable in the book are the lack of any discussion of applications, from which many of the more interesting random walk problems have originated, the absence of any discussion of first-passage time problems, and a description and analysis of having the random walk place in a random, rather than a homogeneous, medium. This has given rise to an enormous amount of research activity, both pure and applied.

George H. Weiss
Center for Information Technology
National Institutes of Health
Bethesda, Md. 20902
E-mail: ghw@helix.nih.gov